# Merrimack School District <br> Mathematics Curriculum 

Kindergarten

## Standards for Mathematical Practice in Kindergarten

The College and Career Readiness Standards for Mathematical Practice are practices expected to be integrated into every mathematics lesson for all students Grades K-12. Below are a few examples of how these Practices may be integrated into tasks that students complete.
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\begin{array}{|l|l|}\hline \text { Practice } & \text { Explanation and Example } \\
\hline \begin{array}{l}\text { 1.Make Sense and } \\
\text { Persevere in } \\
\begin{array}{l}\text { Solving } \\
\text { Problems. }\end{array}\end{array} \begin{array}{l}\text { Mathematically proficient students in Kindergarten begin to develop effective dispositions toward problem solving. In rich } \\
\text { settings in which informal and formal possibilities for solving problems are numerous, young children develop the ability to } \\
\text { focus attention, test hypotheses, take reasonable risks, remain flexible, try alternatives, exhibit self-regulation, and persevere } \\
\text { (Copley, 2010). Using both verbal and nonverbal means, kindergarten students begin to explain to themselves and others the } \\
\text { meaning of a problem, look for ways to solve it, and determine if their thinking makes sense or if another strategy is needed. } \\
\text { As the teacher uses thoughtful questioning and provides opportunities for students to share thinking, kindergarten students } \\
\text { begin to reason as they become more conscious of what they know and how they solve problems. }\end{array} \\
\hline \begin{array}{l}\text { 2.Reason } \\
\text { abstractly and } \\
\text { quantitatively. }\end{array} & \begin{array}{l}\text { Mathematically proficient students in Kindergarten begin to use numerals to represent specific amount (quantity). For } \\
\text { example, a student may write the numeral "11" to represent an amount of objects counted, select the correct number card "17" } \\
\text { to follow "16" on the calendar, or build a pile of counters depending on the number drawn. In addition, kindergarten students } \\
\text { begin to draw pictures, manipulate objects, use diagrams or charts, etc. to express quantitative ideas such as a joining situation } \\
\text { (Mary has 3 bears. Juanita gave her 1 more bear. How many bears does Mary have altogether?), or a separating situation }\end{array}
$$ <br>

(Mary had 5 bears. She gave some to Juanita. Now she has 3 bears. How many bears did Mary give Juanita?). Using the\end{array}\right\}\)| language developed through numerous joining and separating scenarios, kindergarten students begin to understand how |
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| symbols (+, -, =) are used to represent quantitative ideas in a written format. |$|$

$\left.\begin{array}{|l|l|}\hline \begin{array}{l}\text { 5. Use } \\ \text { appropriate } \\ \text { tools } \\ \text { strategically. }\end{array} & \begin{array}{l}\text { In Kindergarten, mathematically proficient students begin to explore various tools and use them to investigate mathematical } \\ \text { concepts. Through multiple opportunities to examine materials, they experiment and use both concrete materials (e.g. 3- } \\ \text { dimensional solids, connecting cubes, ten frames, number balances) and technological materials (e.g., virtual manipulatives, } \\ \text { calculators, and interactive websites) to explore mathematical concepts. Based on these experiences, they become able to } \\ \text { decide which tools may be helpful to use depending on the problem or task. For example, when solving the problem, "There } \\ \text { are 4 dogs in the park. } 3 \text { more dogs show up in the park. How many dogs are in the park?", students may decide to act it out } \\ \text { using counters and a story mat; draw a picture; or use a handful of cubes. }\end{array} \\ \hline \begin{array}{l}\text { 6. Attend to } \\ \text { precision }\end{array} & \begin{array}{l}\text { Mathematically proficient students in Kindergarten begin to express their ideas and reasoning using words. As their } \\ \text { mathematical vocabulary increases due to exposure, modeling, and practice, kindergarteners become more precise in their } \\ \text { communication, calculations, and measurements. In all types of mathematical tasks, students begin to describe their actions } \\ \text { and strategies more clearly, understand and use grade-level appropriate vocabulary accurately, and begin to give precise } \\ \text { explanations and reasoning regarding their process of finding solutions. For example, a student may use color words (such as } \\ \text { blue, green, light blue) and descriptive words (such as small, big, rough, smooth) to accurately describe how a collection of } \\ \text { buttons is sorted. }\end{array} \\ \hline \text { 7. Look for and } \\ \text { make use of } \\ \text { structure }\end{array} \quad \begin{array}{l}\text { Mathematically proficient students in Kindergarten begin to look for patterns and structures in the number system and other } \\ \text { areas of mathematics. For example, when searching for triangles around the room, kindergarteners begin to notice that some } \\ \text { triangles are larger than others or come in different colors- yet they are all triangles. While exploring the part-whole } \\ \text { relationships of a number using a number balance, students begin to realize that } 5 \text { can be broken down into sub-parts, such as } 4 \\ \text { and } 1 \text { or 4 and 2, and still remain a total of 5. }\end{array}\right\}$

## Kindergarten Critical Areas

The Critical Areas are designed to bring focus to the standards at each grade by describing the big ideas that educators can use to build their curriculum and to guide instruction. The Critical Areas for Kindergarten can be found on page 9 in the College and Career Readiness Standards for Mathematics.

1. Representing, relating, and operating on whole numbers, initially with sets of objects.

Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as $5+2=7$ and $7-2=5$. (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.

## 2. Describing shapes and space.

Students describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders, and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

## Grade K Overview

Counting and Cardinality

- Know number names and the count sequence.
- Count to tell the number of objects.
- Compare numbers.

Operations and Algebraic Thinking

- Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

Number and Operations in Base Ten.

- Work with numbers 11 - 19 to gain foundations for place value.


## Measurement and Data

- Describe and compare measureable attributes.
- Classify objects and count the number of objects in categories.

Geometry

- Identify and describe shapes.
- Analyze, compare, create, and compose shapes.

| Counting and Cardinality |  | K.CC |
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| College and Career Readiness Cluster |  |  |
| Know number names and the count sequence. |  |  |
| Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: Introduce written number words zero, one, two...ten (students are not responsible for being able to read these words, but they should be introduced); Know digits and orally count to one hundred |  |  |
| Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: count, object(s), number, in order, sequence, number words, ones, tens, forward |  |  |
| Enduring Understandings: <br> Numbers have names and we can use them to count. |  |  |
|  |  |  |
| Essential Questions: <br> How does counting help us in the real world? How do we count? What are the different ways to count? |  |  |
| College and Career <br> Readiness <br> Standards <br> Students are <br> expected to: | Mathematical Practices | Unpacking Explanations and Examples What does this standard mean that a student will know and be able to do? |
| K.CC.A. 1 Count to 100 by ones and by tens. | K.MP.7. Look for and make use of structure. <br> K.MP.8. Look for and express regularity in repeated reasoning. | Students' rote count by starting at one and counting to 100 When counting by ones, students need to understand that the next number in the sequence is one more. When students count by tens they are only expected to master counting on the decade ( $0,10,20,30,40 \ldots$ ). When counting by tens, students need to understand that the next number in the sequence is "ten more" (or one more group of ten). This objective does not require recognition of numerals. It is focused on the rote number sequence. |


|  |  | Problem Type: Counting Sequence |
| :--- | :--- | :--- |


| count of no objects). | regularity in repeated reasoning. | Due to varied development of fine motor and visual development, reversal of numerals is anticipated. While reversals should be pointed out to students and correct formation modeled in instruction, the emphasis of this standard is on the use of numerals to represent quantities rather than the correct handwriting formation of the actual numeral itself. <br> Students should be given multiple opportunities to count objects and recognize that a number represents a specific quantity. Once this is established, students begin to read and write numerals (numerals are the symbols for the quantities). The emphasis should first be on quantity and then connecting quantities to the written symbols. <br> Examples: <br> - A sample unit sequence might include: <br> 1. Counting up to 30 objects in many settings and situations over several weeks. <br> 2. Beginning to recognize, identify, and read the written numerals, and match the numerals to given sets of objects. <br> 3. Writing the numerals to represent counted objects. <br> Since the teen numbers are not written as they are said, teaching the teen numbers as one group of ten and extra ones is foundational to understanding both the concept and the symbol that represents each teen number. For example, when focusing on the number " 14 ," students should count out fourteen objects using one-to-one correspondence and then use those objects to make one group of ten and four extra ones. Students should connect the representation to the symbol " 14 ." |
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| Counting and Cardinality |  |
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| College and Career Readiness Cluster |  |
| Count to tell the number of objects. |  |
| Students use numbers, including written numerals, to represent quantities and to solve quantitative problems such as counting objects in <br> a set, counting out a given number of objects, and comparing sets or numerals. <br> When learning to count, it is important for kindergarten students to connect the collection of items (4 cubes), the number word ("four"), <br> and the numeral (4), ultimately creating a mental picture of a number. If students simply rote-count a collection of objects without <br> connecting these three components together, they "engage in a meaningless exercise of calling numbers that are one more than the last." <br> (Midget, 2012) Subsidizing, the ability to "instantly see how many" (Clements, 1999), helps students form a mental picture of a number. <br> When students recognize a small collection of objects (e.g., 2 sets of two dots) as one group (e.g., four) - they are beginning to unitize. <br> This ability to see a set of objects as a group is an important step toward being able to see smaller groups of objects within a total <br> collection- which is necessary to decompose number. Materials such as dot cards, dice, and dominoes provide students opportunities to <br> see a variety of patterned arrangements to develop instant recognition of small amounts. <br> Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate <br> mathematical language. The terms students should learn to use with increasing precision with this cluster are: count, object(s), number, <br> number words, number name, forward, pairing, arrange, question |  |
| Enduring Understandings: <br> Everything can be counted. <br> Number names tell us how many objects are in groups and allow us to count in order and compare groups of objects. <br> Essential Questions: <br> Why do we count? |  |
| College and Career <br> Readiness <br> Standards <br> Students are <br> expected to: | Mathematical <br> Practices |
| K.CC.B.4 <br> Understand the <br> relationship <br> between numbers | Unpacking Explanations and Examples <br> What does this standard mean that a student will know and be able to do? <br> abstractly and <br> quantitatively. |


| and quantities; connect counting to cardinality. | K.MP.7. Look for and make use of structure. <br> K.MP.8. Look for and express regularity in repeated reasoning. | Problem Type: Counting <br> This standard focuses on one-to-one correspondence and how cardinality connects with quantity. <br> Example: <br> - When counting three bears, the student should use the counting sequence, "1-2-3," to count the bears and recognize that "three" represents the group of bears, not just the third bear. A student may use an interactive whiteboard to count objects, cluster the objects, and state, "This is three". <br> In order to understand that each successive number name refers to a quantity that is one larger, students should have experience counting objects, placing one more object in the group at a time. <br> Examples: <br> - Using cubes, the student should count the existing group, and then place another cube in the set. Some students may need to re-count from one, but the goal is that they would count on from the existing number of cubes. S/he should continue placing one more cube at a time and identify the total number in order to see that the counting sequence results in a quantity that is one larger each time one more cube is placed in the group. |
| :---: | :---: | :---: |
| K.CC.B. 4 <br> Understand the relationship between numbers and quantities; connect counting to cardinality. <br> a. When counting objects, say the number names in the standard order, pairing each object | K.MP.2. Reason abstractly and quantitatively. K.MP.7. Look for and make use of structure. <br> K.MP.8. Look for and express regularity in repeated reasoning. | Students implement correct counting procedures by pointing to one object at a time (one-toone correspondence), using one counting word for every object (synchrony/ one-to-one tagging), while keeping track of objects that have and have not been counted. This is the foundation of counting. |


| with one and only <br> one number name <br> and each number <br> name with one and <br> only one object. |  |  |
| :--- | :--- | :--- |
| K.CC.B.4 <br> Understand the <br> relationship <br> between numbers <br> and quantities; <br> connect counting <br> to cardinality. | K.MP.2. Reason <br> abstractly and <br> quantitatively. <br> K.MP.7. Look for <br> and make use of <br> structure. | Students answer the question "How many are there?" by counting objects in a set and <br> understanding that the last number stated when counting a set (...8, 9, 10) represents the total <br> amount of objects: "There are 10 bears in this pile." Since an important goal for children is to <br> count with meaning, it is important to have children answer the question, "How many do you <br> have?" after they count. Often times, children who have not developed cardinality will count <br> the amount again, not realizing that the 10 they stated means 10 objects in all. |
| b. Understand that <br> the last number <br> name said tells the <br> number of objects <br> counted. The <br> number of objects <br> is the same <br> regardless of their <br> arrangement or the <br> order in which <br> they were counted. | Young children believe what they see. Therefore, they may believe that a pile of cubes that <br> regularity in <br> repeated reasoning. | they counted may be more if spread apart in a line. As children move towards the <br> developmental milestone of conservation of number, they develop the understanding that the <br> number of objects does not change when the objects are moved, rearranged, or hidden. <br> Children need many different experiences with counting objects, as well as maturation, <br> before they can reach this developmental milestone. |
| K.CC.B.4 <br> Understand the <br> relationship <br> between numbers <br> and quantities; <br> connect counting <br> to cardinality. | K.MP.2. Reason <br> abstractly and <br> quantitatively. <br> K.MP.7. Look for <br> and make use of <br> structure. | Another important milestone in counting is inclusion (aka hierarchal inclusion). Inclusion is <br> based on the understanding that numbers build by exactly one each time and that they nest <br> within each other by this amount. For example, a set of three objects is nested within a set of <br> 4 objects; within this same set of 4 objects is also a set of two objects and a set of one. Using <br> this understanding, if a student has four objects and wants to have 5 objects, the student is <br> able to add one more- knowing that four is within, or a sub-part of, 5 (rather than removing <br> all 4 objects and starting over to make a new set of 5). This concept is critical for the later <br> development of part/whole relationships. |


| c. Understand that <br> each successive <br> number name <br> refers to a quantity <br> that is one larger. | K.MP.8. Look for <br> and express <br> regularity in <br> repeated reasoning. | Students are asked to understand this concept with and without (0-30) objects. For example, <br> after counting a set of 8 objects, students answer the question, "How many would there be if <br> we added one more object?"; and answer a similar question when not using objects, by <br> asking hypothetically, "What if we have 5 cubes and added one more. How many cubes <br> would there be then?" |
| :--- | :--- | :--- |
| K.CC.B.5 Count <br> to answer "how <br> many?" questions <br> about as many as <br> 30 things arranged <br> in a line, a <br> rectangular array, <br> or a circle, or as <br> many as 10 things <br> in a scattered <br> configuration; <br> given a number <br> from 1-30, count <br> out that many <br> objects. | K.MP.2. Reason <br> abstractly and <br> quantitatively. <br> K.MP.7. Look for <br> and make use of <br> structure. | K.MP.8. Look for <br> and express <br> regularity in <br> repeated reasoning. |
| Keeping track is a method of counting that is used to count each item once and only once <br> when determining how many. After numerous experiences with counting objects, along with <br> the developmental understanding that a group of objects counted multiple times will remain <br> the same amount, students recognize the need for keeping track in order to accurately <br> determine "how many". |  |  |
| Depending on the amount of objects to be counted, and the students' confidence with <br> counting a set of objects, students may move the objects as they count each, point to each <br> object as counted, look without touching when counting, or use a combination of these <br> strategies. It is important that children develop a strategy that makes sense to them based on <br> the realization that keeping track is important in order to get an accurate count, as opposed to <br> following a rule, such as "Line them all up before you count", in order to get the right <br> answer. |  |  |

## Examples:

- If items are placed in a circle, the student may mark or identify the starting object.
- If items are in a scattered configuration, the student may move the objects into an organized pattern.
- Some students may choose to use grouping strategies such as placing objects in twos, fives, or tens (note: this is not a kindergarten expectation).
- Counting up to 30 objects should be reinforced when collecting data to create charts and graphs.

| Counting and Cardinality |
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| College and Career Readiness Cluster |
| Compare numbers |
| Students should develop a strong sense of the relationship between quantities and numerals before they begin comparing numbers. |
| Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate <br> mathematical language. The terms students should learn to use with increasing precision with this cluster are: greater, more, less, fewer, <br> equal, same amount, compare |
| Enduring Understandings: <br> Sets of objects can be named, grouped, and counted so that we can compare them in terms of greater than, less than, or equal to. <br> Essential Questions: <br> How can we compare numbers? <br> How can comparing numbers help you understand their value? |

\begin{tabular}{|c|c|c|c|c|}
\hline College and Career Readiness Standards Students are expected to: \& Mathematical Practices \& \multicolumn{3}{|l|}{Unpacking Explanations and Examples What does this standard mean that a student will know and be able to do?} \\
\hline \begin{tabular}{l}
K.CC.C. 6 \\
Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. \({ }^{1}\)
\end{tabular} \& \begin{tabular}{l}
K.MP.2. Reason abstractly and quantitatively. \\
K.MP.7. Look for and make use of structure. \\
K.MP.8. Look for and express regularity in repeated reasoning.
\end{tabular} \& \multicolumn{3}{|l|}{Students use their counting ability to compare sets of objects (0-10). They may use matching strategies (Student 1), counting strategies (Student 2) or equal shares (Student 3) to determine whether one group is greater than, less than, or equal to the number of objects in another group.} \\
\hline \({ }^{1}\) Include groups with up to ten objects. \& \& \begin{tabular}{l}
Student 1 \\
I lined up one square and one triangle. Since there is one extra triangle, there are more triangles than squares. \\
\(\triangle\) \\
\(\square\) \\
\(\square\)

 \& 

Type: Comparing <br>
Student 2 <br>
I counted the squares and I got 4. Then I counted the triangles and got 5. Since 5 is bigger than 4, there are more triangles than squares.

 \& 

bers <br>
Student 3 <br>
I put them in a pile. I then took away objects. Every time I took a square, I also took a triangle. When I had taken almost all of the shapes away, there was still a triangle left. That means that there are more triangles than squares.
\end{tabular} <br>

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\end{tabular}

$\left.\begin{array}{|l|l|l|}\hline & & \begin{array}{l}\text { Examples: } \\ -\quad \text { Matching: Students use one-to-one correspondence, repeatedly matching one object from } \\ \text { one set with one object from the other set to determine which set has more objects. }\end{array} \\ \text { - Observation: Students may use observation to compare two quantities (e.g., by looking at } \\ \text { two sets of objects, they may be able to tell which set has more or less without counting). } \\ \text { - Observations in comparing two quantities can be accomplished through daily routines of } \\ \text { collecting and organizing data in displays. Students create object graphs and pictographs } \\ \text { using data relevant to their lives (e.g., favorite ice cream, eye color, pets, etc.). Graphs } \\ \text { may be constructed by groups of students as well as by individual students. }\end{array}\right\}$

## Operations and Algebraic Thinking <br> K.OA <br> College and Career Readiness Cluster <br> Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from. <br> For numbers $0-10$, Kindergarten students choose, combine, and apply strategies for answering quantitative questions. This includes quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away. Objects, pictures, actions, and explanations are used to solve problems and represent thinking. Although College and Career Readiness Standards - Mathematics states, "Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten in encouraged, but it is not required", please note that it is not until First Grade when "Understand the meaning of the equal sign" is an expectation (1.OA.7). <br> Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: join, add, putting together, taking apart, taking from, subtract, and, same amount as, equal to, less than, more than, total, count on, count all, equations, minus, total. <br> Note on vocabulary: The term "total" is used here instead of the term "sum." "Sum" sound the same as "some," but has the opposite meaning. "Some" is used to describe problem situations with one or both addends unknown, so it is better in the earlier grades to use "total" rather than "sum." Formal vocabulary for subtraction ("minuend" and "subtrahend") is not needed for Kindergarten, Grade 1, and Grade 2, and may inhibit students seeing and discussing relationships between addition and subtraction. At these grades, the terms "total" and "addend" are sufficient for classroom discussion.

## Enduring Understandings:

Addition is putting groups together.
Subtraction is taking groups apart.

## Essential Questions:

What happens when we combine groups?
What happens when we take groups apart?

| College and Career Readiness Standards Students are expected to: | Mathematical Practices | Unpacking Explanations and Examples What does this standard mean that a student will know and be able to do? |
| :---: | :---: | :---: |
| K.0A.A.1. <br> Represent <br> addition and <br> subtraction with <br> objects, fingers, <br> mental images, <br> drawings, sounds <br> (e.g., claps), <br> acting out <br> situations, verbal <br> explanations, <br> expressions, or equations. <br> (Drawings need not show details, but should show the mathematics in the problems. <br> This applies <br> wherever <br> drawings are mentioned in the Standards.) | K.MP.1. Make sense of problems and persevere in solving them. <br> K.MP.2. Reason abstractly and quantitatively. <br> K.MP.4. Model with mathematics. <br> K.MP.5. Use appropriate tools strategically. | Students demonstrate the understanding of how objects can be joined (addition) and separated (subtraction) by representing addition and subtraction situations in various ways. This objective is focused on understanding the concept of addition and subtraction, rather than reading and solving addition and subtraction number sentences (equations). Using addition and subtraction in a word problem context allows students to develop their understanding of what it means to add and subtract. <br> College and Career Readiness Standards for Mathematics states, "Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required." Please note that it is not until First Grade when "Understand the meaning of the equal sign" is an expectation (1.OA.7). Therefore, before introducing symbols (,,$+-=$ ) and equations, kindergarteners require numerous experiences using joining (addition) and separating (subtraction) vocabulary in order to attach meaning to the various symbols. For example, when explaining a solution, kindergartens may state, "Three and two is the same amount as 5." While the meaning of the equal sign is not introduced as a standard until First Grade, if equations are going to be modeled and used in Kindergarten, students must connect the symbol $(=)$ with its meaning (is the same amount/quantity as). <br> Examples: Students should use objects, fingers, mental images, drawing, sounds, acting out situations and verbal explanations in order to develop the concepts of addition and subtraction. Then, they should be introduced to writing expressions and equations using appropriate terminology and symbols which include " + ," ",- " and " $=$ ". <br> - Addition terminology: add, join, put together, plus, combine, total <br> - Subtraction terminology: minus, take away, separate, difference, compare |



| K.0A.A.3. <br> Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., $5=$ $2+3$ and $5=4+$ 1). | K.MP.1. Make sense of problems and persevere in solving them. <br> K.MP.2. Reason abstractly and quantitatively. <br> K.MP.4. Model with mathematics. <br> K.MP.7. Look for and make use of structure. <br> K.MP.8. Look for and express regularity in repeated reasoning. | Students develop an understanding of part-whole relationships with number pairs which add to a specified total, 1-10. These number pairs may be examined either in or out of context. Thus, when breaking apart a set (decompose), students use the understanding that a smaller set of objects exists within that larger set (inclusion). <br> In Kindergarten, students need ample experiences breaking apart numbers and using the vocabulary "and" \& "same amount as" before symbols $(+,=)$ and equations $(5=3+2)$ are introduced. If equations are used, a mathematical representation (picture, objects) needs to be present as well. <br> Example: Bobby Bear is missing 5 buttons on his jacket. How many ways can you use blue and red buttons to finish his jacket? Draw a picture of all your ideas. <br> Students could draw pictures of: <br> 4 blue and 1 red button <br> 3 blue and 2 red buttons <br> 2 blue and 3 red buttons <br> 1 blue and 4 red buttons <br> Students may use objects such as cubes, two-color counters, square tiles, etc. to show different number pairs for a given number. For example, for the number 5, students may split a set of 5 objects into 1 and 4,2 and 3 , etc. <br> Students may also use drawings to show different number pairs for a given number. For example, students may draw 5 objects, showing how to decompose in several ways. |
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| K.0A.A.4. For <br> any number from <br> 1 to 9, find the <br> number that <br> makes 10 when <br> added to the <br> given number, <br> e.g., by using <br> objects or <br> drawings, and <br> record the answer | K.MP.1. Make <br> sense of problems <br> and persevere in <br> solving them. |
| K.MP.2. Reason <br> abstractly and <br> quantitatively. <br> K.MP.4. Model <br> with mathematics. <br> K.MP.7. Look for <br> and make use of <br> structure. |  |

Sample unit sequence:

- A contextual problem (word problem) is presented to the students such as, "Mia goes to Nan's house. Nan tells her she may have 10 pieces of fruit to take home. There are lots of apples and bananas. How many of each can she take?"
- Students find related number pairs using objects (such as cubes or two-color counters), drawings, and/or equations. Students may use different representations based on their experiences, preferences, etc.
- Students may write equations that equal 10 such as:

$$
\begin{array}{ll}
\circ & 5=4+1 \\
\circ & 3+2=5 \\
\circ & 2+3=4+1
\end{array}
$$

This is a good opportunity for students to systematically list all the possible number pairs for a given number. For example, all the number pairs for 5 could be listed as $0+5,1+4,2+3,3+2,4+1$, and $5+0$. Students should describe the pattern that they see in the addends, e.g., each number is one less or one than the previous addend.
Students build upon the understanding that a number (less than or equal to 10) can be decomposed into parts (K.OA.3) to find a missing part of 10. Through numerous concrete experiences, kindergarteners model the various sub-parts of ten and find the missing part of 10 .

## Example:

When working with 2-color beans, a student determines that 4 more beans are needed to make a total of 10 .

"I have 6 beans. I need 4 more beans to have 10 in all."
In addition, kindergarteners use various materials to solve tasks that involve decomposing and composing 10 .


|  |  | Example: The student snaps ten cubes together to make a "train." <br> - Student breaks the "train" into two parts. S/he counts how many are in each part and record the associated equation $(10=$ $\qquad$ $+$ _). $\qquad$ <br> - Student breaks the "train into two parts. S/he counts how many are in one part and determines how many are in the other part without directly counting that part. Then $\mathrm{s} / \mathrm{he}$ records the associated equation (if the counted part has 4 cubes, the equation would be 10 $=4+$ ). $\qquad$ <br> - Student covers up part of the train, without counting the covered part. S/he counts the cubes that are showing and determines how many are covered up. Then s/he records the associated equation (if the counted part has 7 cubes, the equation would be $10=7+$ $\qquad$ ). <br> Example: The student tosses ten two-color counters on the table and records how many of each color are facing up. |
| :---: | :---: | :---: |
| K.0A.A.5. <br> Fluently add and subtract within 5 . | K.MP.2. Reason abstractly and quantitatively. <br> K.MP.7. Look for and make use of structure. <br> K.MP.8. Look for and express regularity in repeated reasoning. | This standard focuses on students being able to add and subtract numbers correctly within 5 . Adding and subtracting fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently without resorting to counting. <br> Students develop fluency by understanding and internalizing the relationships that exist between and among numbers. Oftentimes, when children think of each "fact" as an individual item that does not relate to any other "fact", they are attempting to memorize separate bits of information that can be easily forgotten. Instead, in order to fluently add and subtract, children must first be able to see sub-parts within a number (inclusion, K.CC.4.c). <br> Once they have reached this milestone, children need repeated experiences with many different types of concrete materials (such as cubes, chips, and buttons) over an extended amount of time in order to recognize that there are only particular sub-parts for each number. Therefore, children will realize that if 3 and 2 is a combination of 5 , then 3 and 2 cannot be a combination of 6 . <br> After making various arrangements with toothpicks, students learn that only a certain number of sub-parts exist within the number 4 : |


|  |  | Then, after numerous opportunities to explore, represent and discuss " 4 ", a student becomes able to fluently answer problems such as, "One bird was on the tree. Three more birds came. How many are on the tree now?" and "There was one bird on the tree. Some more came. There are now 4 birds on the tree. How many birds came?" <br> Traditional flash cards or timed tests have not been proven as effective instructional strategies for developing fluency. Rather, numerous experiences with breaking apart actual sets of objects and developing relationships between numbers help children internalize parts of number and develop efficient strategies for fact retrieval. <br> Strategies students may use to attain fluency include: <br> - Counting on (e.g., for $3+2$, students will state, " 3 ," and then count on two more, " 4,5 ," and state the solution is " 5 ") <br> - Counting back (e.g., for $4-3$, students will state, " 4 ," and then count back three, " $3,2,1$ " and state the solution is " 1 ") <br> - Counting up to subtract (e.g., for 5-3, students will say, "3," and then count up until they get to 5, keeping track of how many they counted up, stating that the solution is " 2 ") <br> Using doubles (e.g., for $2+3$, students may say, "I know that $2+2$ is 4 , and 1 more is 5 ") <br> - Using commutative property (e.g., students may say, "I know that $2+1=3$, so $1+2=3$ ") <br> - Using fact families (e.g., students may say, "I know that $2+3=5$, so $5-3=2$ ") |
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## Numbers and Operations in Base Ten <br> K.NBT <br> College and Career Readiness Cluster

Work with numbers 11-19 to gain foundations for place value.
Rather than unitizing a ten (recognizing that a set of 10 objects is a unit called a "ten"), which is a standard for First Grade (1.NBT.1a), kindergarteners keep each count as a single unit as they explore a set of 10 objects and leftovers.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: number words (one, two... thirteen, fourteen, ... nineteen), left over, ones, ten frame

## Enduring Understandings:

We can compose and decompose numbers to help us understand their value.
Knowing the value of numbers in each place will help us add and subtract.

## Essential Questions:

How does composing and decomposing numbers in groups of tens and ones help us to understand their value?

| College and <br> Career Readiness <br> Standards <br> Students are <br> expected to: | Mathematical <br> Practices | Unpacking Explanations and Examples <br> What does this standard mean that a student will know and be able to do? |
| :--- | :--- | :--- |
| K.NBT.A.1. <br> Compose and <br> decompose <br> numbers from 11 <br> to 19 into ten <br> ones and some <br> further ones, e.g., <br> by using objects <br> or drawings, and | K.MP.1. Make <br> sense of problems <br> and persevere in <br> solving them. | Students explore numbers 11-19 using representations, such as manipulatives or drawings. Keeping <br> K.MPstractly and <br> quantitatively. |
| each count as a single unit, kindergarteners use 10 objects to represent "10" rather than creating a <br> unit called a ten (unitizing) as indicated in the First Grade College and Career Readiness <br> Standard 1.NBT.1a: 10 can be thought of as a bundle of ten ones - called a "ten." |  |  |
| Example: <br> Teacher: "I have some chips here. Do you think they will fit on our ten frame? Why? Why Not?" <br> Students: Share thoughts with one another. <br> Teacher: "Use your ten frame to investigate." |  |  |



| Measurement and Data |  | K.MD |
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| College and Career Readiness Cluster |  |  |
| Describe and compare measurable attributes. |  |  |
| Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: length, weight, heavy(ier), light(er), long(er), big(ger), small(er), more of, most, least, less of, longer, taller, shorter, similarities, differences, alike, different, compare |  |  |
| Enduring Understanding: <br> Objects can be described using different units of measurements in order to compare. |  |  |
| Essential Questions: <br> How can we describe and compare the length of objects? How can we describe and compare the weight of objects? |  |  |
| College and Career Readiness Standards Students are expected to: | Mathematical Practices | Unpacking Explanations and Examples <br> What does this standard mean that a student will know and be able to do? |
| K.MD.A.1. <br> Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object. | K.MP.7. Look for and make use of structure. | In order to describe attributes such as length and weight, students must have many opportunities to informally explore these attributes. Students describe measurable attributes of objects, such as length, weight, and size. <br> Example: A student may describe a shoe with one attribute, "My shoe is heavy!" or more than one attribute, "This shoe is heavy! It's also really long." <br> Example: When describing a soda can, a student may talk about how tall, how wide, how heavy, or how much liquid can fit inside. <br> These are all measurable attributes. Non-measurable attributes include: words on the object, colors, pictures, etc. |

$\left.\begin{array}{|l|l|l|}\hline & & \begin{array}{l}\text { Students often initially hold undifferentiated views of measurable attributes, saying that one object is } \\ \text { "bigger" than another whether it is longer, or greater in area, or greater in volume, and so forth. } \\ \text { Example: Two students might both claim their block building is "the biggest." Conversations about }\end{array} \\ \begin{array}{l}\text { how they are comparing- one building may be taller (greater in length) and another may have a }\end{array} \\ \text { larger base (greater in area) help students learn to discriminate and name these measureable } \\ \text { attributes. As they discuss these situations and compare objects using different attributes, they learn } \\ \text { to distinguish, label, and describe several measureable attributes of a single object. Thus, teachers } \\ \text { listen for and extend conversations about things that are "big", or "small," as well as "long," "tall," } \\ \text { or "high," and name, discuss, and demonstrate with gestures the attribute being discussed. }\end{array}\right\}$

|  |  | As students develop conservation of length, learning and using language such as "It looks longer, <br> but it really isn't longer" is helpful. <br> When making direct comparisons for length, students must attend to the "starting point" of each |
| :--- | :--- | :--- |
| object. For example, the ends need to be lined up at the same point, or students need to compensate |  |  |
| when the starting points are not lined up (conservation of length includes understanding that if an |  |  |
| object is moved, its length does not change; an important concept when comparing the lengths of |  |  |
| two objects). |  |  |
| Language plays an important role in this standard as students describe the similarities and |  |  |
| differences of measurable attributes of objects (e.g., shorter than, taller than, lighter than, the same |  |  |
| as, etc.). |  |  |


| count. (Limit category counts to be less than or equal to 10 ). |  | Then the student counts the number of buttons in each pile: blue (5), green (4), orange (3), purple (4). <br> Finally, the student organizes the groups by the quantity. "I put the purple buttons next to the green buttons because purple also had (4). Blue has 5 and orange has 3 . There aren't any other colors that have 5 or 3 . So they are sitting by themselves." <br> This objective helps to build a foundation for data collection in future grades as they create and analyze various graphical representations. <br> Possible objects to sort include buttons, shells, shapes, beans, etc. After sorting and counting, it is important for students to: <br> - Explain how they sorted the objects <br> - Label each set with a category <br> - Answer a variety of counting questions that ask, "How many ..." and compare sorted groups using words such as, "most", "least", "alike" and "different" |
| :---: | :---: | :---: |


| Geometry |
| :--- |
| College and Career Readiness Cluster |
| Identify and describe shapes (square, circle, triangle, rectangle, hexagon, cube, cone, cylinder, and sphere). |
| Students will understand that certain attributes define what a shape is called (number of sides, number of angles, etc.) and other attributes <br> do not coolor, size, orientation). Using geometric attributes, the student identifies and describes squares, circles, triangles, rectangles, <br> hexagons, cubes, cones, cylinders, and spheres. Throughout the year, Kindergarten students move from informal language to describe what <br> shapes look like (e.g.,."That looks like an ice cream cone!") to more formal mathematical language (e.g., "That is a triangle. All of its sides <br> are the same length". <br> In Kindergarten, students need ample experiences exploring various forms of the shapes (e.g., size: big and small; types: triangles, <br> equilateral, isosceles, scalene; orientation: rotated slightly to the left, 'upside down') using geometric vocabulary to describe the different <br> shapes. <br> Students in Kindergarten typically recognize figures by appearance alone, often by comparing them to a known example of a shape, such as <br> the triangle on the left (see below). For example, students in Kindergarten typically recognize that the figure on the left as a triangle, but <br> claim that the figure on the right is not a triangle, since it does not have a flat bottom. Thus, the properties of a figure are not recognized or <br> know. Students typically make decisions on identifying and describing shapes based on perception, not <br> reasoning. |
| Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical <br> language. The terms students should learn to use with increasing precision with this cluster ares squares, circles, triangles, rectangles, <br> hexagons, cubes, cones, cylinders, sheres, flat solid, side, corner, angle, edge, face, positional vocabulary (e.g., above, below, beside, <br> in front of, behind, next to, same, different, etc.). |
| Enduring Understanding: <br> Objects can be described and compared using their geometric attributes. |
| Essential Questions: <br> What are the different shapes in our world? <br> What are the different parts of a shape? |


| College and Career Readiness Standards Students are expected to: | Mathematical Practices | Unpacking Explanations and Examples <br> What does this standard mean that a student will know and be able to do? |
| :---: | :---: | :---: |
| K.G.A.1. <br> Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to. | K.MP.7. Look for and make use of structure. | Students locate and identify shapes in their environment. For example, a student may look at the tile pattern arrangement on the hall floor and say, "Look! I see squares! They are next to the triangle." At first students may use informal names e.g., "balls," "boxes," "cans". Eventually students refine their informal language by learning mathematical concepts and vocabulary and identify, compare, and sort shapes based on geometric attributes. <br> Students also use positional words (such as those italicized in the standard) to describe objects in the environment, developing their spatial reasoning competencies. Kindergarten students need numerous experiences identifying the location and position of actual two-and-three-dimensional objects in their classroom/school prior to describing location and position of two-and-threedimension representations on paper. <br> Examples of environments in which students would be encouraged to identify shapes would include nature, buildings, and the classroom using positional words in their descriptions. <br> Teachers should work with children and pose four mathematical questions: Which way? How far? Where? And what objects? To answer these questions, children develop a variety of important skills contributing to their spatial thinking. <br> Examples: <br> - Teacher holds up an object such as an ice cream cone, a number cube, ball, etc. and asks students to identify the shape. Teacher holds up a can of soup and asks," What shape is this can?" Students respond "cylinder!" <br> - Teacher places an object next to, behind, above, below, beside, or in front of another object and asks positional questions. Where is the water bottle? (water bottle is placed behind a book) Students say "The water bottle is behind the book." |


| K.G.A.2. <br> Correctly name <br> shapes regardless <br> of their <br> orientations or <br> overall size. | K.MP.7. Look for <br> and make use of <br> structure. | Through numerous experiences exploring and discussing shapes, students begin to understand that <br> certain attributes define what a shape is called (number of sides, number of angles, etc.) and that <br> other attributes do not (color, size, orientation). As the teacher facilitates discussions about shapes <br> ("Is it still a triangle if I turn it like this?"), children question what they "see" and begin to focus <br> on the geometric attributes. <br> Kindergarten students typically do not yet recognize triangles that are turned upside down as <br> triangles, since they don't "look like" triangles. Students need ample experiences manipulating <br> shapes and looking at shapes with various typical and atypical orientations. Through these <br> experiences, students will begin to move beyond what a shape "looks like" to identifying <br> particular geometric attributes that define a shape. <br> Students should be exposed to many types of triangles in many different orientations in order to <br> eliminate the misconception that a triangle is always right-side-up and equilateral. |
| :--- | :--- | :--- |
| K.G.A.3 Identify <br> shapes as two- <br> dimensional or <br> three dimensional | K.MP.7. Look for <br> and make use of <br> structure. | Students should also be exposed to many shapes in many different sizes. <br> Examples: |
| Teacher makes pairs of paper shapes that are different sizes. Each student is given one <br> shape and the objective is to find the partner who has the same shape. |  |  |
| Teacher brings in a variety of spheres (tennis ball, basketball, globe, ping pong ball, etc.) to |  |  |
| demonstrate that size doesn't change the name of a shape. |  |  |


| Geometry |  | K.G |
| :---: | :---: | :---: |
| College and Career Readiness Cluster |  |  |
| Analyze, compare, create, and compose shapes |  |  |
| Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: compare, compose, attributes, sides, vertices/corners, vertex, two-and three-dimensional, same, different |  |  |
| Enduring Understanding: <br> Objects can be similar to others in one way and different in other ways. <br> Essential Questions: <br> How are shapes the same? How are shapes different? |  |  |
| College and Career Readiness Standards Students are expected to: | Mathematical Practices | Unpacking Explanations and Examples What does this standard mean that a student will know and be able to do? |
| K.G.B.4. Analyze <br> and compare twoand threedimensional <br> shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/ "corners") and other attributes | K.MP.6. Attend to precision. <br> K.MP.7. Look for and make use of structure. | Students analyze and compare two- and three-dimensional shapes by observations. Their visual thinking enables them to determine if things are alike or different based on the appearance of the shape. Students sort objects based on appearance. Even in early explorations of geometric properties, they are introduced to how categories of shapes are subsumed within other categories. For instance, they will recognize that a square is a special type of rectangle. <br> Students should be exposed to triangles, rectangles, and hexagons whose sides are not all congruent. They first begin to describe these shapes using everyday language and then refine their vocabulary to include sides and vertices/corners. Opportunities to work with pictorial representations, concrete objects, as well as technology helps student develop their understanding and descriptive vocabulary for both two- and three- dimensional shapes. Kindergarteners also distinguish between the most typical examples of a shape from obvious non-examples. <br> Example: When comparing a triangle and a square, they note that they both are closed figures, have straight sides, but the triangle has 3 sides while the square has 4 . Or, when building in the Block Center, they notice that the faces on the cube are all square shapes. |

$\left.\begin{array}{|l|l|l|}\hline \begin{array}{l}\text { (e.g., having sides } \\ \text { of equal length). }\end{array} & \begin{array}{l}\text { Example: When identifying the triangles from a collection of shapes, a student circles all of the } \\ \text { triangle examples from the non-examples }\end{array} \\ \hline \begin{array}{l}\text { K.G.B.5 Model } \\ \text { shapes in the } \\ \text { world by building } \\ \text { shapes from } \\ \text { components (e.g., } \\ \text { sticks and clay } \\ \text { balls) and drawing } \\ \text { shapes. }\end{array} & \begin{array}{l}\text { K.MP.1. Make } \\ \text { sense of problems } \\ \text { and persevere in } \\ \text { solving them. } \\ \text { K.MP.4. Model } \\ \text { with mathematics. }\end{array} & \begin{array}{l}\text { Students apply their understanding of geometric attributes of shapes in order to create given shapes. } \\ \text { Because two-dimensional shapes are flat and three-dimensional shapes are solid, students may draw } \\ \text { or build two-dimensional shapes and only build three-dimensional shapes. Shapes could be built } \\ \text { using materials such as clay, toothpicks, marshmallows, gumdrops, straws, pipe cleaners, etc. } \\ \text { Students should understand and identify two-dimensional shapes used to construct three- } \\ \text { dimensional shapes. } \\ \text { and make use of } \\ \text { structure. }\end{array}\end{array} \begin{array}{l}\text { Example: Students may roll a clump of play-doh into a sphere or use their finger to draw a triangle } \\ \text { in the sand table, recalling various attributes in order to create that particular shape. }\end{array}\right\}$

|  | K.MP.4. Model <br> with mathematics. <br> MP.7. Look for <br> and make use of <br> structure. | Students also combine shapes to build pictures. They first use trial and error (part a) and gradually <br> consider components (part b). | Combining shapes to build pictures |
| :--- | :--- | :--- | :--- |



| Standards for Mathematical Practices (MP) |  |  |
| :---: | :---: | :---: |
| Standards <br> Students are expected to: | Mathematical Practices <br> Mathematical <br> Practices are listed throughout the grade level document in the 2nd column to reflect the need to connect the mathematical practices to mathematical content in instruction. | Explanations and Examples |
| K.MP.1. Make sense of problems and persevere in solving them. |  | In Kindergarten, students begin to build the understanding that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Younger students may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" or they may try another strategy. |
| K.MP.2. Reason abstractly and quantitatively. |  | Younger students begin to recognize that a number represents a specific quantity. Then, they connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities. |
| K.MP.3. Construct viable arguments and critique the reasoning of others. |  | Younger students construct arguments using concrete referents, such as objects, pictures, drawings, and actions. They also begin to develop their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking. |


| K.MP.4. Model with <br> mathematics. |  | In early grades, students experiment with representing problem situations <br> in multiple ways including numbers, words (mathematical language), <br> drawing pictures, using objects, acting out, making a chart or list, <br> creating equations, etc. Students need opportunities to connect the <br> different representations and explain the connections. They should be <br> able to use all of these representations as needed. |
| :--- | :--- | :--- |
| K.MP.5. Use appropriate <br> tools strategically. |  | Younger students begin to consider the available tools (including <br> estimation) when solving a mathematical problem and decide when <br> certain tools might be helpful. For instance, kindergarteners may decide <br> that it might be advantageous to use linking cubes to represent two <br> quantities and then compare the two representations side-by-side. |
| K.MP.6. Attend to <br> precision. | As kindergarteners begin to develop their mathematical communication <br> skills, they try to use clear and precise language in their discussions with <br> others and in their own reasoning. |  |
| K.MP.7. Look for and <br> make use of structure. | Younger students begin to discern a pattern or structure. For instance, <br> students recognize the pattern that exists in the teen numbers; every teen <br> number is written with a 1 (representing one ten) and ends with the digit <br> that is first stated. They also recognize that 3 + 2 = 5 and 2 + 3 = 5. |  |
| K.MP.8. Look for and <br> express regularity in <br> repeated reasoning. |  | In the early grades, students notice repetitive actions in counting and <br> computation, etc. For example, they may notice that the next number in a <br> counting sequence is one more. When counting by tens, the next number <br> in the sequence is "ten more" (or one more group of ten). In addition, <br> students continually check their work by asking themselves, "Does this <br> make sense?" |

